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ABSTRACT

This action research describes a program for improving mathematical problem solving skills. The targeted population consisted of first grade students in a transient, middle class community as well as third and sixth grade students from a growing, middle to upper class in Illinois. The concerns of problem solving were documented through teacher input and classroom observations. When analyzing the probable cause data, it was revealed that teachers were not consistent when implementing the problem solving skills needed for success across grade levels. It has also been shown that students lacked the ability to self-monitor and apply a variety of problem solving strategies to mathematical tasks. Instead, students opted to use basic computational skills to solve complex mathematical problems. A review of solution strategies resulted in a choice of three inventions: a consistency among teachers throughout grade levels, a four step problem solving checklist for students, and an incorporation of self-monitoring strategies within the classroom. Post intervention data indicated an increase in student use and understanding of problem solving strategies and self monitoring check list. (Contains 37 references.) (Author)

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BEYOND COMPUTATION: IMPROVING MATHEMATICAL PROBLEM SOLVING

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Abstract

This action research describes a program for improving mathematical problem solving skills. The targeted population consisted of first grade students in a transient, middle class community as well as third and sixth grade students from a growing, middle to upper class community in Illinois. The concerns of problem solving were documented through teacher input and classroom observations.

When analyzing the probable cause data, it was revealed that teachers were not consistent when implementing the problem solving skills needed for success across grade levels. It has also been shown that students lacked the ability to self-monitor and apply a variety of problem solving strategies to mathematical tasks. Instead, students opted to use basic computational skills to solve complex mathematical problems.

A review of solution strategies resulted in a choice of three interventions: a consistency among teachers throughout grade levels, a four step problem solving checklist for students, and an incorporation of self-monitoring strategies within the classroom.

Post intervention data indicated an increase in student use and understanding of problem solving strategies and a self monitoring check list.

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CHAPTER 1

PROBLEM STATEMENT AND CONTEXT

General Statement of the Problem

Mathematical problem solving is an ongoing technique that is used throughout one's lifetime. The students of the targeted first, third, and sixth grades lacked a variety of skills needed to be successful mathematical problem solvers. Evidence for the existence of the problem included results from standardized and teacher-made tests, teacher surveys, and recorded teacher observations.

Immediate Problem Context

Site A

Site A had a total enrollment of 518 students. About 95% of Site A's student population was White. The remainder was composed of 2.1% Asian/Pacific Islander, 1.5% Black, and 1.2% Hispanic. Furthermore, 1.4% of the student population came from low-income families. The attendance rate was stable at 95.8% with a 4.8% mobility rate. Last, limited English proficiency students encompassed 2.1% of the population (State School Report Card, 1999).

Site A had a total of 42 teachers. Of this total, the average teaching experience was 13.4 years. An average of 39.2% of the teachers had a bachelor's degree while 60.8% had obtained a master's degree or above. The district's teacher racial/ethnic background was 97% White, 1.2% Black, 1.2% Hispanic, .6% Asian/Pacific Islander and 0% Native American. The faculty was 87.7% female and 12.3% male. The average teacher's salary was $45,167, while the average administrator's salary was $79,832 (State School Report Card, 1999).

Site A was a fairly new facility located in an expanding suburban community. The building was established in 1993. While the building was designated as a kindergarten through second grade facility, recent over-population had led to the addition of two third grade classrooms. The school was equipped with a library, computer lab, and computer equipped classrooms.

Surrounding the school was a large sports field as well as a student playground. While the location was in the heart of a subdivision, most of the students used public transportation.

In the past two years, Site A adopted a new language arts and mathematics curriculum. Following, it also adopted a new science curriculum. Programs have evolved to a more traditional approach due to parental requests as well as reforms to comply with current state testing standards. Site A was in the process of developing an accelerated mathematics program.

Site O

Site O had a total enrollment of 776 students. About 92% of Site O's student population was White. The remainder was composed of 2.7% Asian/Pacific Islander, 2.6% Black, 1.8% Hispanic, finally .4% Native American. Furthermore, 4.8% of the student population came from low-income families. Continually, the attendance rate was stable at 96% with a 2.2% mobility rate. Last, limited English proficiency students encompassed 1.1% of the population (State School Report Card, 1999).

Site O had a total population of 45 teachers. Of this total, the average teaching experience was 13.4 years. An average of 39.2% of the teachers had a bachelor's degree while 60.8% had obtained a master's degree or above. The district's teacher racial/ethnic background was 97% White, 1.2% Black, 1.2% Hispanic, .6% Asian/Pacific Islander, and 0% Native American. The faculty was 87.7% female and12.3% male. The average teacher's salary was $45,167, while the average administrator's salary was $79,832 (State School Report Card, 1999).

Site O was a 4 year old middle school located at a major connecting road. There were no sidewalks around the building. Because of the location, all of the students had to ride a bus or obtained private transportation to Site O. The academic half of the building was three stories high. Additionally, two gyms, a cafeteria, and special program classrooms filled the other half of the building's one story section. Finally, Site O was highly technologically equipped with a computer lab, three computers per classroom as well as television monitors in every classroom.

The math program had recently gone through a revision. New textbooks were adopted as well as an honors program. Furthermore, attempts were made to accelerate the rate at which the students were expected to achieve their next educational level.

Site W

Site W had a total enrollment of 716 students. About 73% of Site W's student population was White. The remainder was composed of 19.7% Black and 6.8% Hispanic. Furthermore, 22.8% of the student population came from low-income families. Finally, the attendance rate was stable at 94.2% with an 18.2% mobility rate. Limited English proficiency students encompass 2.4% of the population.

Site W had a total faculty of 54 teachers. Of this total, the average teaching experience was 16.6 years. An average of 72.4% of the teachers had a bachelor's degree while 27.6% had a master's degree or above. The district's teacher racial/ethnic background was 100% White. The faculty was 96.3% female and 3.7% male. The average teacher's salary was $39,299 while the average administrator's salary was $71,895 (State School Report Card, 1999).

Site W's building was a combination of both original and remodeled facility. While originally the building was designated for kindergarten through third grade students, at the time of the research it was used for grades kindergarten through sixth grade. The classrooms and library were technologically equipped with several computers. The school was surrounded by a retention pond, play areas, and parking areas.

Site W was located in the center of the community. The students were provided with bus transportation, walked to school, or were dropped off by other means.

Math was constantly being reevaluated due to low standardized test scores. More time was being spent on the math curriculum. After school programs were also developed and low achieving students were assigned to peer math tutors.

The Surrounding Community

Site A and Site O

Site A and Site O were located in the same suburban community whose population was approximately 47, 600 residents. The median age of residents was 36.7 years of age with a median family income of $89,708. Within the community population, 93.8% were White, 2.4% were Hispanic, .3% were Black, and 3.5% were of other nationalities. The median years of school completed was 13.8 years. Furthermore, the percentage of employment for those over the age of 16 was 67.6% (Local Tribune, 2000).

The community had a student population of 5,551 distributed among ten school buildings.

A total of seven elementary and three middle schools fed into one neighboring high school. The community's population was rapidly increasing due to the development of new housing projects throughout the area. Parental involvement was evident within the school structure. With a 99.7% rate of parent to teacher contact, schools were constantly receiving parent input and feedback on the district's curriculum (State School Report Card, 1999).

One pressing issue affecting the school community was the distribution of students according to the present district boundary lines. Within the past 5 years many new homes had been built in southwest areas of the community. With new families and students moving in, the boundary lines had not been changed to reflect equal population within the surrounding school buildings. Due to this, many buildings in the southwest areas had experienced over-crowding while others had a lower enrollment as well as a smaller class size.

Another important issue within the district was the push for an accelerated program within the curriculum. The district was in the process of creating an accelerated mathematics program at the elementary level. This included self-contained classrooms as well as clusters within the regular education system. In addition, at the middle school level, administrators were implementing an accelerated mathematics program that offered self-contained honors classes.

Site W

Site W was in a changing community with a population of approximately 11,200 residents. The median age of residents was 33 years of age with a median family income of $57,171. Within the community population, 79.2% were White, 14.4% were Black, 4.6% were Hispanic, and 1.7% were of other descent. The median years of school completed was 12.6 years. Furthermore, the percentage of employment for those over the age of 16 was 72.4% (State School Report Card, 1999).

Site W was in a community whose student population was distributed among four school buildings; three elementary schools, and one junior high school. Recently, the community's population increased due to some land acquisition by construction companies. Parental involvement was evident within the school structure. Parents came into the school to volunteer with parties, assist students in writing stories, act as guest readers, and chaperone at field trips. With a 100% rate of parent-to-teacher contact, the school was constantly providing opportunities

for parents to be involved in school activities, parent-teacher conferences, after school programs, and telephone conversations (State School Report Card, 1999).

An issue that affected the school was the constant preparation for school improvement by the staff of Site W. As a result, school improvement planning committees had been formed to provide direction toward practicing math, reading, language, and science. In addition, Site W implemented "homework help" as an after school homework hour.

Another issue affecting Site W was the high level of mobility. Students were provided with fragmented information regarding the curriculum. This directly affected the student's ability to connect information after they transferred from school to school.

National Context of Problem

Mathematical problem solving is an ongoing process that begins as early as preschool and is carried throughout one's lifetime. Burns (1995) suggested five major goals to reform mathematics for all students:

1. students learn to value mathematics
2. students become confident in their ability to do mathematics
3. students become mathematical problem solvers
4. students learn to communicate mathematically
5. students learn to reason mathematically. (p. 5)

Knowing this, one can see that mathematics is evolving and a need for improvement of problem solving is evident. According to Higgins (1997), "Students receiving problem-solving instruction showed more perseverance in solving problems, more positive attitudes, and more sophisticated definitions of mathematical understanding" (p. 13).

The issue of problem solving has generated concern both in the national and local school settings based on recent state test scores. Recent interviews with teachers concluded that, "students and teachers believe that problem solving is primarily an application of computational skills" (Ford, 1988, p. 315). Knowing this, it is important that students have the means to decide what type of computational problem they are solving. This often requires a higher level of problem solving skills.

This general concern for improved problem solving skills has led to revisions in standardized testing. Students are required to demonstrate mathematical abilities that extend beyond basic computational problems. All of these changes focus on the need for an increase in the level of problem solving abilities.

A basic understanding of problem solving has a lifelong impact on students' success. Understanding the steps involved in solving a problem will improve further education, group involvement, and future workplace environments. According to this, schools need to address the issue of instruction in the problem solving process for improved student success.

CHAPTER 2

PROBLEM DOCUMENTATION

Problem Evidence

In order to document the lack of mathematical problem solving skills in the targeted sites, a teacher survey was given, student pretests were administered, and periodic student interviews were conducted at each site. A letter was sent home to parents explaining the project before it began (Appendix A). Parents agreed to have their children participate in this study.

The teachers were given a survey at the beginning of the study (Appendix B). This survey recorded their attitudes and feelings about mathematical problem solving. This survey also allowed teachers to provide input as to how they incorporate mathematical problem solving within their own classrooms.The questions asked teachers to evaluate how often they incorporate vocabulary, key concepts, and group interaction when teaching mathematics. The survey also asked teachers to evaluate how often they apply mathematics in real-world situations. For example, when teaching estimation, an instructor might discuss a real world situation that involves estimating money amounts at the grocery store. Another example might involve story problems that require students to estimate time restraints necessary to complete certain tasks. The survey goes on to ask whether or not teachers allow students to solve mathematical problems in a variety of ways. For example, for a given problem, one child may reach a solution by drawing a picture or diagram, another may solve with the use of manipulatives, while still another may solve the problem numerically. All students reach the same answer, but all used a different method. What the teacher-reasearchers want to gain input about is how often or not this occurs in classrooms. By allowing a variety of solutions, students are given the freedom to explore mathematically while making conscious and logical decisions. The final survey question asked instructors to determine how often students give either written or verbal solutions to mathematical problems. Fifty teachers participated in the survey. The results of the survey are presented in two parts. Table 1 reflects

teachers' practical applications of problem solving. Table 2 reflects general concerns that teachers have regarding adequate means for teaching problem solving skills on a daily basis.

Table 1

Teachers' Responses to Survey on Mathematical Problem Solving

Topics questioned	% Never	% Seldom	% Sometimes	% Often	% Always
1. use of math vocabulary	0	6	72	20	2
2. real-world application	0	0	44	54	2
3. self-monitoring checklist	24	34	30	4	8
4. group discussion	0	14	24	38	24
5. mastery of key concepts	0	6	42	43	6
6. use a variety of applications	0	16	36	38	10
7. written or verbal solutions	0	24	28	40	8

Table 1 illustrates the finding of the survey. Table 1 shows that almost three fourths of the teachers sometimes required their students to use mathematical vocabulary and language. This fraction demonstrating that vocabulary is only being used sometimes is a probable indication that students are lacking in the mathematical terminology they need to problem solve. According to Morgan and Richardson, (2000) "The main reason for vocabulary study is to develop concepts and help students see relationships inherent in what they are reading. Teachers need to take sufficient time to prepare for the lesson by sometimes having students study difficult vocabulary terms" (p. 274). If this margin increased, teachers would most likely see that there is a direct correlation between the comprehension of vocabulary terms and the ability to break apart and solve mathematical problems.

The survey also revealed that 43% of teachers surveyed revolved their mathematics curriculum around memorization of concepts. This conflicts with today's shift among mathematics research and experts. According to Goldsmith (1999):

> New views of mathematical standards present a view of mathematics learning, teaching, and assessment that shifts the focus of curriculum and instruction. Whereas traditional mathematics education focuses on memorization, rote learning, and the application of facts

> and procedures, the Standards-based approach emphasizes the development of conceptual understanding and reasoning. (p. 40)

Memorizing basic facts and concepts does not necessarily promote the use of higher level thinking skills, but as the survey suggests, a high percentage of instructors base much of their curriculum around the memorization of basic facts.

While a majority of the teachers surveyed said they often use memorization and vocabulary when teaching, 38% said they often use a variety of applications for solving a problem. This low percentage demonstrates that many instructors are not allowing students to explore a variety of mathematical options. "Children can learn important mathematical ideas when they have opportunities to engage in solving a variety of problems. Children should also have many opportunities to talk or write about how they solve problems while teachers should elicit childrens' thinking" (Carpenter et al., 1996, p. 403). In many instances teachers demonstrate "the correct method" for reaching a solution. This may be helpful to some children, but for others who see other possibilities for solving the problem, this immediately limits their ability to express their knowledge and it can often inhibit them from future exploration.

Table 2

Teachers' Responses to Survey on Concerns Dealing with Mathematical Problem Solving

Concerns questioned	% Yes
1. time concerns	62
2. problem solving materials provided	42
3. reflection journals used	36
4. benefit from continuing education	100

Table 2 shows that 62% of teachers surveyed feel time is a concern when teaching problem solving in the classroom. The teacher researchers agreed with this percentage due to their own classroom scheduling. So often teachers are tied to a limiting slot of available time for mathematics due to the growing need to cover other necessary areas of study. Elementary school teachers often need to cover up to seven subject areas within a given day, while teachers at the junior high and high school level are faced with a 40 to 45 block of time for teaching mathematics. This

results in teachers spending the majority of their allotted time on basic skills while leaving only a small amount of time to focus on problem solving. Teachers at times understand a need for problem solving, however, the ever-expanding curriculum does not allocate enough time to fit all of the instruction. Also, 58% feel the curriculum does not provide enough problem solving material, while 42% feel they have adequate materials. Based on the materials available, the teacher researchers noted that within the mathematics curriculum, teachers are provided with daily problem solving activities, a variety of manipulatives, problem solving suggestions, and sections throughout the provided text manuals. This poses the concern of whether these materials are being implemented. There is a strong possibility that many teachers are not aware of the abundance of materials they have available and that they also lack the skills needed to utilize them in a daily classroom setting.

Finally, 100% of the teachers surveyed reported they would benefit from workshops pertaining to problem solving. This shows the desire teachers have to improve their knowledge of teaching problem solving in the classroom. "There is greater recognition today at the local, state, and national levels that sustained, high-quality staff development is essential if all students are to achieve at high levels" (Hirsch & Sparks, 1997, p. 1) This gives hope that administrators will recognize a growing need to provide staff-development opportunities for their teachers in the area of mathematical problem solving.

Another method of documenting the problem at the targeted sites was a teacher administered pretest consisting of five problem solving problems (Appendix C). The problems focused on a variety of strategies. The results of each site are listed in Figures 1, 2 and 3.

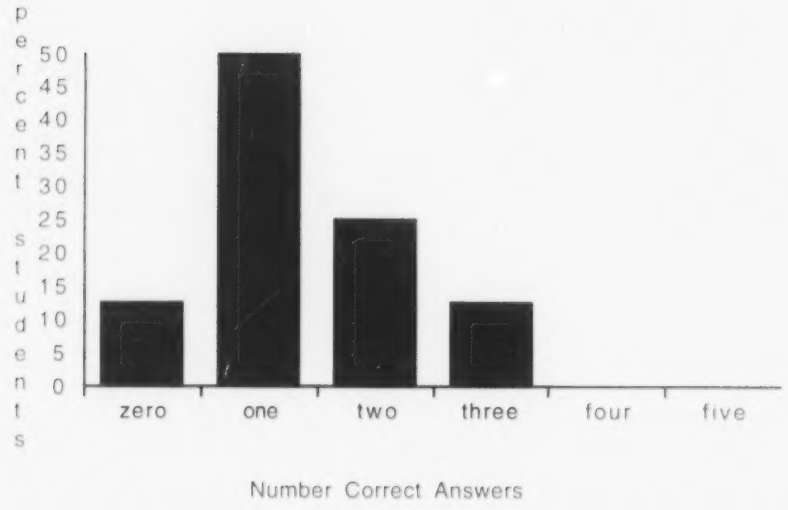

Figure 1. Site A pretest results of percent students and number of correct answers on problem solving task.

Figure 1 clearly demonstrates a lack of mathematical problem solving abilities among the students. Half of the children only achieved one correct answer on the pretest, while none of the students received more than three correct responses.

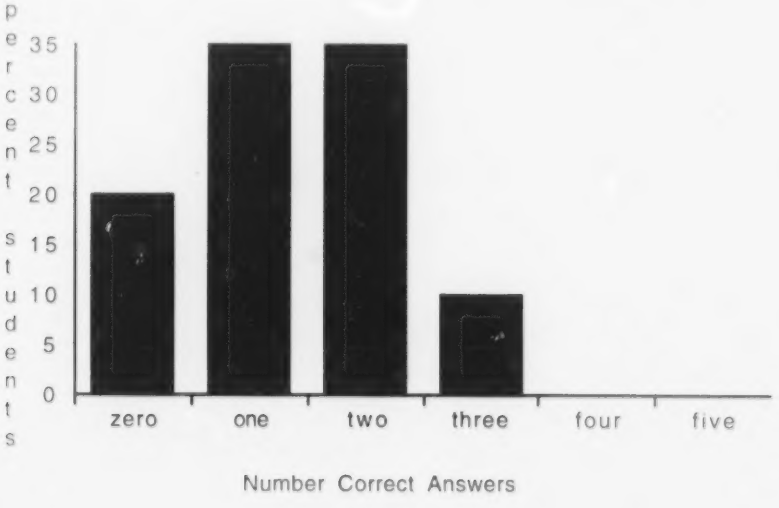

Figure 2. Site O pretest results of percent students and number of correct answers on problem solving task.

The results in Figure 2 show 90% of the students answered two or fewer questions correctly. This figure suggests that students lack problem solving skills within their classrooms based on the low number of correct responses

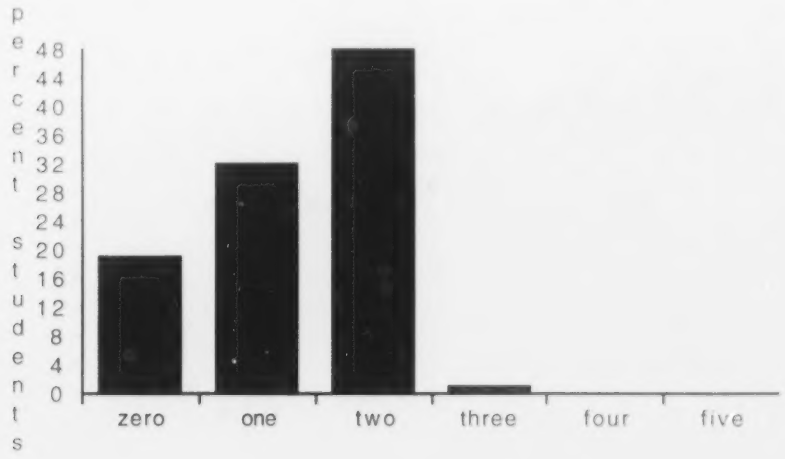

Figure 3. Site W pretest results of percent students and number of correct answers on problem solving task.

Finally, Figure 3 corresponds with the results obtained in Figures 1 and 2. Only 1% of the children received three correct answers, while again, the majority of the class performed in the zero to two correct range.

Overall, Figures 1, 2, and 3 support the idea the students today are lacking the necessary skills for mathematical problem solving. The next section will generate some probable causes for this ever-growing concern.

Probable Causes

Mathematical problem solving skills are needed not only in a classroom setting, but, they may also prove to be very useful in real-life situations. Estimating, patterning, and constructing an organized list accompanied with basic mathematical skills are tools that every one needs to function in their adult life. What is happening in many classrooms, however, is that many children are being taught their basic skills without ever having to apply them in everyday problem solving situations. This lack of connection leads students to look at mathematics as separated from the real

world. The literature reviewed suggested a variety of causes that contribute to this lack of problem solving skills among students.

Marginal Emphasis

One such cause for this is that many of the skills and strategies necessary for successful problem solving are simply not being taught or applied within the classroom curriculum. Teachers today are faced with many difficulties and obstacles when attempting to incorporate these skills. With a demanding curriculum it is often difficult enough to find the time to teach children basic math skills. Many teachers adhere to teaching basic skills within their allotted math time and they often graze over problem solving. This means that while students may sometimes see mathematical problem solving in their textbooks, they are not necessarily given the time to explore, expand, and solve this type of mathematical material within their math schedule. "Problem solving is not an easy topic to teach to elementary students. At the very age that they are most dependent on concrete representations of all concepts, we ask them to adopt an abstract general approach to problems" (McCoy, 1994, p. 1).

Because problem solving generates mathematical difficulties for children early on, it is often overlooked by elementary instructors who claim that teachers really need to devote their teaching time to fundamental math concepts. This in turn creates more difficulties for teachers in higher grade levels. Many teachers opt to teach from a more traditional standpoint due to state and district demands for increased standardized testing scores. Instead of focusing on methods, instructors strive to assess the knowledge of basic skills and fact memorizations. According to Thompson (1982), studies in mathematics education indicate that teachers' beliefs about mathematics, about learning, and about teaching affect the way in which mathematics is taught.

One can conclude from this that instruction within a mathematical classroom is likely to vary from classroom to classroom. How mathematics is taught is, therefore, based on an individual teacher's interpretations of the goals and content within his or her mathematics curriculum. This in turn leads to gaps across grade levels. The National Assessment of Educational Progress (as cited in Bhat et al., 1998) indicated that word problems in mathematics present difficulties for students of all ages and ability levels. The report demonstrated that the majority of U.S. students perform at a much lower level in mathematical problem solving than students in other industrialized countries.

After reviewing this comparison, the National Council of Teachers of Mathematics (as cited in Schoenfeld, 1985) expressed:

> at the heart of their recommendations and most other calls for reform in mathematics education is the view that current elementary school mathematics curriculum overemphasizes efficient computational arithmetic skill at the expense of understanding problem solving. Most researchers and mathematics educators agree that there is more to mathematics than computational proficiency. But beyond this agreement exists diverse views about what it means to know, understand, and learn mathematics. (p. 5)

Once teachers realize that problem solving skills are a necessity in a mathematics classroom, they also need to demonstrate a consistency within grade levels and across the curriculum.

Inconsistent Teaching

A second probable cause for a lack of problem solving skills is that teachers are not consistent with what they are actually teaching. While one teacher may devote a great deal of time on problem solving, another may only touch on it from time to time, while still another may never touch on it at all. Most evident is the fact that teachers are often not clear on what problem solving is, as well as the knowledge of how to successfully teach it. When interviewing a group of fifth grade teachers, it was reported that the "teachers believed that problem solving is primarily an application of computational skills, while students' beliefs are, for the most part, consistent with the beliefs held by the teacher" (Ford, 1994, p. 314). This reveals that students often misunderstand what problem solving involves based upon their instructors' personal feelings and attitudes towards problem solving. One can also see how this would cause confusion for children traveling to various teachers and various grade levels. Without being taught a consistent method, students are only left with fragments of the problem solving skills they need.

One particular study completed in an elementary school demonstrated how children deal with specific problem solving skills. The study concluded:

> Students will try imaginatively to deal with problems and will usually follow their process and construct a solution, even though it may be incorrect because of 'bugs'. One source of these 'bugs' is that when children begin school they often have considerable

> informal mathematics experience and partial understandings of many mathematical concepts which form inaccurate and/or incomplete schema for solving problems. (Maurer, 1987, p. 266)

The Maurer study suggested while students may not grasp every problem solving concept immediately, they do have the capacity to attain this knowledge if it is properly taught beginning at an early age. Teachers beginning as early as kindergarten can begin a problem solving background that can be carried out and advanced in future grade levels. The opposite of this occurs when teachers ignore the need to problem solve, and leave children to build on their own misconceptions of what problem solving is. Along with some misconceptions pertaining to the meaning of problem solving comes the problem of exploring it in a variety of settings. Math textbooks and teacher manuals provide instructors with problems and solutions. However, in the classroom setting, students often veer from textbook answers in the attempt to discover their own meaning. By doing so students are developing strategies without even being aware of it. This can work beautifully for a child if it is recognized, but often the opposite occurs. Teachers will stick to one "correct" strategy and require that all of the students use the strategy for the given problems.

There is also a feeling of disconnection within mathematics when students are unable to build connections between mathematics and their own lives. When instructors insist on one devised strategy, students are unable to take ownership of the solution. According to Araujo, Cassunde, and Schliemann (1998):

> Schools, as an important cultural setting in children's lives, play an essential role in the development of mathematical knowledge. The variety of situations that can be set up in classrooms provide opportunities for learning new and more powerful procedure for understanding aspects of mathematical content that may not easily appear out of school. (p. 422)

Providing mathematical experiences that have personal meaning will help children better develop their own strategies.

Lack of Direct Teaching of Math Vocabulary

Related to inconsistent teaching is a third probable cause. This cause is the lack of mathematical vocabulary or poor mathematical vocabulary of students. This cause relates to inconsistent teaching due to the fact that many educators do not teach students the mathematical

vocabulary they need to solve problems. When students are asked to solve a word or story problem many can often come up with a solution with little or no prompting. What is lacking, however, is the follow-up that explains how the solution was obtained. What educators need to conceptualize is that much of the math language that needs to be taught is not within a child's daily usage. Therefore, it should not be expected that children will automatically pick up on unfamiliar math terms. Like teaching key words in reading, repetition is the approach needed to obtain mastery. According to Morgan and Richardson (2000) "The constant repetition of definitions is a good reinforcer for the aural learner" (p. 273). These authors also go on to say that, "Teachers need to take sufficient time to prepare for a reading lesson by sometimes having students study difficult vocabulary terms before reading" (p. 279). In this respect, new mathematical terms will overlap into the realm of reading and should be addressed in much the same fashion as reading vocabulary. Unfortunately, this is often not the case. Instead, instructors emphasize the answers to problem solving questions while skimming over valuable and often helpful mathematical language.

For those students who have difficulties discovering a strategy or picking out key words, a vocabulary bank would be extremely beneficial. "Too often, unique math vocabulary is not explained, such as divisors, integers, quotients, multiples, differences, products, multiplicands, minuends, and subtrahends, and other math specific terms. Yet, they abound in textbooks and are used frequently in word problems" (Cornell, 1999, p. 2). With explanations of terms, children develop a better chance of reading a problem and solving its meaning. Mathematics is a language and as such has a grammar. There is a grammar for problem solving. "One cannot communicate well with written or spoken words if the grammar of the language is not used. In the same way one cannot communicate effectively, with others and one's self, through problem solving without using the grammar of problem solving" (McAllister, 1997, p. 3).

Lack of Metacognitive and Reflective Practice

A fourth cause for a lack of problem solving skills involves a lack of time to discuss the problems as well as providing opportunities for students to self-monitor their solutions. Mathematics is a curriculum that requires teachers to touch on a large number of topics in a very limited amount of time. Knowing this, it is often difficult to devote the time needed to not only discuss vocabulary and strategies, but to allow students to reflect on their own solutions and ideas.

Too often teachers simply explain answers and move on due to a lack of time. "For most of us there is a direct correlation between time and learning: the more time, the more learning" (Hope, 1999, p. 3). Classrooms need to devote more time to the process and steps that are used to discover a solution rather than just the answer itself. Students also need to evaluate their own work to discover their strengths and weaknesses. Often children are asked to proofread and evaluate their writing, but within mathematics there is little time to write reflections of proofread solutions. Many instructors worry about giving a large block of writing time within mathematics. "Teachers are concerned that more writing will not be the best use of the limited time. During math, children need to do many other things. The need to construct, sort, read, experiment , interact, create, and think" (Fuqua, 1996, p.1). Knowing teachers have these feelings, it is apparent that many instructors are not willing to lend writing time within a math curriculum. Providing time for explanations will lead to a deeper meaning instead of just finding an answer and moving on.

Besides a lack of written explanation within mathematics, there is also a lack of oral expression. Teachers concern themselves with covering an extensive amount of material leaving little time for explanation and exploration. Teachers are apt to pose a question without allowing students "think time" before expecting a response. Frakes and Klein (2000) suggest:

> It is important to be patient! Young mathematicians need time to develop these qualities. With more experience, exposure, guidance, and role modeling from the teacher, students will become more engaged, more assertive, and better able to listen to others and make decisions about whether they agree or disagree. (p. 379)

If educators can slow down, students will have more opportunities to think.

Lack of Self-Monitoring

Students also lack in their abilities to use and develop their own thinking strategies when devising solutions to a given problem. Many students reach answers without having the knowledge to demonstrate how they developed them. This is due to an absence of self-monitoring. "Students' deficiencies in using self-monitoring while learning have been attributed to motivational factors" (Chyung et al., 1998). The students might be uninformed or misinformed about the effectiveness of self-monitoring. They, therefore, may lack the desire to use self-monitoring strategies. If educators model self-monitoring and allow the time for it, students will

begin to comprehend the benefits of self-monitoring and in turn they will reap the rewards of a stronger mathematical background. "In the classroom, content-area teachers can begin focusing on authentic assessment by keeping a checklist of essential developments that they want students to demonstrate. At this level the students' development-trial and error dialogue, self-criticism can be assessed more readily" (Morgan & Richardson, 2000, p. 236). With a developed checklist, both teachers and students will more readily have the means for achieving solutions.

The need to place a stronger emphasis on mathematical problem solving is evident based on the following causes: (a) strategies are not being taught or applied, (b) lack of connection throughout the curriculum, (c) lack of or poor mathematical vocabulary, and (d) little or no time to self monitor. With a stronger mathematical problem solving background, students will have the ability to take what they know and apply it in real world situations.

CHAPTER 3

THE SOLUTION STRATEGY

Literature Review

In the quest to correct the mathematical problem solving dilemma, experts provide many possibilities that would aid students in becoming competent mathematical problem solvers. Most obvious in this situation is the need to define for both educators and students what mathematical problem solving actually is. Educators will then have the ability to break down strategies and make them clear for student comprehension. "Problem solving is the process by which students experience the power and usefulness of mathematics in the world around them" (Neil, 1999, p. 66).

The key word in the given definition is experience. If educators merely glaze over mathematical problem solving material, students are not actually given the opportunity to experience what problem solving is. By giving children the time to explore and develop their own questions and solutions, educators will be providing real mathematical experiences with which students will readily grasp and understand. When reviewing a mathematical definition, problem solving entails more than just the knowledge of basic facts. In a similar definition, mathematical problem solving is described as "the set of actions that need to be taken to perform the task of solving a problem" (Virva, 1980, p. 5). While the second is a somewhat antiquated definition, it is in many ways similar to that of the National Council of Teachers of Mathematics (NCTM). Both express the need to focus on process rather than product when considering mathematical problem solving. Teachers need to understand that problem solving within a mathematics curriculum is a step-by-step process that deserves and requires an extended amount of time and attention. Along with this educators also need to focus on the way in which a given problem is solved. If we are to look at process first, then we also need to accept the fact that there is often more than on way to develop a solution. Problem solving is a process that should permeate throughout the mathematics program and should be the context in which concepts and skills are learned (Neil, 1996). More

important than the simple recognition of what mathematical problem solving means, is the growing need to incorporate it on a daily basis. The constant representation will provide meaning to those problems that in the past were only looked at for answers.

Direct Teaching of Metacognition

Once there is a knowledge base for the meaning of mathematical problem solving, the next step is to implement these skills and strategies within the mathematics curriculum. One of the best ways to begin this process is by giving students the time to think about and reflect on their own thinking. This type of instruction includes time for discussion as well as time for self-evaluation and reflection. Weaver (1985) states the following:

> Some of the best learning about problem solving may occur after the problem solution has been attained. It is important to think about how a problem is solved. In fact, research indicates that time spent discussing and reconsidering their thinking may be more important than any other strategy in helping children become better problem solvers. (p. 40)

Students often start in kindergarten with many opportunities to explore and expand their knowledge when using various manipulatives and generating their own mathematical stories and problems. Kindergarten children are also encouraged to draw pictures and symbols to help them reach their desired goals in math. However, as the grade level increases it often seems just when exploration time should increase, the opposite usually occurs. For many upper grades, math class is designed to provide calculated answers using a paper and pencil. Students are discouraged when using their fingers or writing extensive answers due to their lack of time. Once the answer is supplied the teacher moves on to the next problem and the discussion is, at best, minimal. This rote calculation can be useful, but in many instances teachers will not accept discussion or argument during math instruction simply because it is supposed to be understood that there is only one correct response and only one way to reach it. "Too often, children are given problems for which there is one and only one correct solution. Almost all textbook problems are like that" (Linquist, Reys, & Suydam, 1992, p. 39). By veering away from these paper and pencil textbook problems educators can help children think on a more independent level rather than constantly having to rely on calculations.

One possible suggestion for expanding students abilities to think about their strategies is to generalize. Teachers should spend time breaking down the content of story problems in order to

look for and discuss similarities and differences among them. By doing this students will being to pick out key terms and phrases and see how they relate to specific strategies. "Analyzing the structural features of a problem rather than focusing only on details often results in insights more significant than the answer to the specific situation posed in the problem" (Lindquist, Reys, & Suydam, 1992, p. 39).

Another useful strategy to aid student thought processes is attempting to change the child's point of view about a given problem. When devising a plan of attack for a particular problem students will quickly adopt their own point of view and they will also make certain assumptions. If their plan turns out to be unsuccessful, students have an extremely difficult time seeing the problem from a different standpoint. Instead, children will often continue to follow their original scheme, leaving them again and again with the incorrect solution. Teachers need to help students to redefine their criteria for the problem so that this type of frustration does not continually occur. Students need to look at the problem more closely to examine what it precisely says. They also need to disregard their own assumptions and should never imply anything that is not being directly stated within the context of the problem. By doing this, the focus goes back to the problem rather than the answer allowing students more opportunity to explore the facts and data.

Student metacognition can also greatly improve simply by requiring the students to go back and check their solutions. It can be acceptable for an instructor to grade basic skills tests and not allow students to go back and change their answers. With problem solving, though, the goal is to improve and expand a child's thought processes. As stated by Lindquist, Reys, and Suydam (1992):

> Checking has long been advocated as a way to help children pinpoint their errors-provided they do not simply make the solution and check agree. One way of checking is going through the procedures again. Another is verifying the reasonableness of the answer: is it a plausible answer to the question posed in the problem? (p. 39)

If mistakes are marked incorrect, but never reviewed, students will not comprehend their own errors and they will likely make the same errors again. By reviewing a problem step by step, students will better understand their errors and be less likely to make the same mistakes on similar problems in the future.

Checklists and Self-Assessment

Having the ability to think through problems independently greatly improves a child's sense of mathematics and it allows them to grow and develop throughout the grade levels. Accompanied with a structured thought process is the implementation of tools, checklists, and reflections that will enhance understanding and organization. Giving students a problem without any sense of structure or direction lead to confusion and disarray. This is why teachers should provide their students with a devised plan that will work for all mathematical problems. One such strategy that has proven to be successful is a four-step problem solving method developed by the authors of Middle School Math (Bolster, et al., 1999). In their 1999 mathematics series, the authors of Middle School Math suggested the implementation of the following four steps:

1. Do I understand the problem?
2. Did I develop a plan?
3. Now I can solve the problem.
4. Look back.

While simple in nature, these four steps give students an opportunity to think through a plan and devise a solution before they record an answer to a given problem. Within the context of these four steps exists key questions that will not allow students to move on before they fully understand what a problem requires and what strategies are necessary for solving it. It is suggested that educators not only discuss these four steps, but, also devise a checklist that students can use when problem solving. This checklist can prove to be extremely useful for those students who tend to rush through a mathematical problem without fully comprehending what they are being asked to do.

Teachers may wish to use their own inventories and checklists to evaluate problem solving strategies among students. An inventory can be used to check what a student knows about problem solving strategies. An instructor might give students one of several problems and ask them to solve each with a specified strategy or to solve each using two to three specified strategies. The aim, of course, is to find out whether or not the student can apply each strategy, not what the answer to the problem is. Checklists may also be used to record individualized information about students' needs understandings, attitudes, and content achievement. This supplies the instructor with valuable information when attempting to help students become better problem solvers. A

checklist will pinpoint specific areas of need when working through problems and teachers will be more readily available to address areas of problem solving that need attention.

Journal writing in mathematics can be an effective tool for evaluating progress as well. Many instructors view writing as a separate area of concentration so it is often overlooked in the area of mathematics. Mathematical journals can be used to assess strategies as well as give students the chance to record some of their thoughts and feelings about problems solving. "Journals give a means for children to tangibly represent their finding, both as a record for themselves and to show others how they solved a problem (Fuqua, 1997, p. 73). Instead of correcting papers and sending them home to be lost or thrown away, journal entries muck like checklists and inventories, can be used as an ongoing problem solving record that can be referred to when questions arise. Students then have the opportunity to share their written and planned out solutions with other students without having to simply discuss from memory. The journal entries provide written documentation that can easily be brought to a small group or whole class discussion. "Like most worthwhile endeavors, helping students produce journal writing that will effect their understanding of mathematics and illustrate their feeling and knowledge of mathematics will require time and patience. It will also require practice on the students part" (Norwood, 1994, p. 147). While this may begin as a long and frustrating task, the end results will benefit both the teacher and students.

Real-World Application

Another strategy that is useful to implement into the mathematics curriculum is the use of problems that encourage real-world applications. Oftentimes children view their mathematical studies as separate from what goes on in their everyday lives. By developing thinking strategies that involve real-world concepts, students are able to place value on what is being taught. Not only will they be able to design strategies and solutions to problems, but they will also be able to make connections to their own lives. Students' informal mathematical ideas cannot be ignored. "These informal ideas, along with previously learned formal ideas, form the current experiential mathematical background for students" (Battista, 1994, p. 1788). Teachers need to examine the lifestyles and make-up of the classroom to develop appropriate mathematical problem solving material that can be utilized within the class. All classrooms vary so, of course, this type of

instruction will need to change with each new class. As stated by Araujo, Cassunde, and Schliemann (1998):

> Schools are an important cultural setting in children's lives and they play an essential role in the development of mathematical knowledge. The variety of situations that can be set up in classrooms provides opportunities for learning new and more powerful procedures and for understanding aspect of mathematical content that may not easily appear out of school. (p. 422)

Real-world problems arise in a variety of modes, offering many opportunities for teaching. Teachers need to make use of problems posed by the children or by situations in which the classroom finds itself. Frakes and Klein (2000) suggested teachers apply mathematical ideas to other events in their lives and make these connections spontaneously. The ability to connect mathematical concepts to, and apply them in, the real world is an important attribute of all mathematicians. "Effective math instruction can be enhanced by increasing real-world applications, integrating projects and contests to generate more interest in immediate correction of student errors to ensure continuous learning" (Cornell, 1999, p. 255). Students take a much more involved interest in situations related directly to themselves. Bringing in problem solving games that present good problem solving situations where students can discuss and use a variety of strategies is motivating. Personalizing problems can also be an interest booster for children. "Simply substituting the names of children in the classroom within problem situations can help many children to accept problems that otherwise would seem remote or uninteresting" (Lindquist, Reys, & Suydam, 1992, p. 40). Making these minor changes to already existing mathematical materials will lead in increased involvement and participation. Students will experience the enjoyment of a challenge while developing a positive self image as well as a positive image of the other children around them. Daniels, Hyde, and Zemelan (1993) state:

> Students should be encouraged to formulate and solve problems directly related to the world around them so they can see the structures of mathematics in every aspect of their lives. Concrete experiences and materials provide the foundation for understanding concepts and constructing meaning. Students must truly create their own way of interpreting an idea, relating it to their own personal life experiences, seeing how it fits with what they already know, and how they are thinking about related ideas. (p. 73)

Making these types of connections with mathematical story problems ensures student interest will increase and productivity will also improve.

Teacher Training and Staff Development

Moving away from the actual processes, strategies, and skills needed to be successful in mathematical problem solving takes us to the role of the educator. For many teachers, the process of teaching mathematical problem solving instead of the basic skills requires teachers to change their mindset about the nature of mathematics. This would encompass a teacher's beliefs about what it means to learn and teach mathematics. When teachers change their focus on mathematical thinking they go beyond teaching skills about mathematics, and begin to view it as a problem solving and making sense endeavor. "Studies on mathematics teachers' knowledge, beliefs, attitudes, practices, and professional development are forming a growing body of literature about the mathematics teacher" (Chapman, 1999, p.121). The focus of much of the literature mentioned in the Chapman article expressed a need for educators to gain perspective about how mathematics has evolved. While the shift from traditional teaching may be challenging for some teachers, such a shift is necessary in order for mathematics instruction to be valuable across grade levels.

One solution that will ease the shift away from traditional methods is the opportunity for teachers to become familiar with mathematical problem solving through inservice and classes provided by their school districts. In order to use many of these approaches to mathematics, teachers must think in ways substantially different from how many of them were taught about students, subject matter, and the teaching and learning process.

While teachers may be enthusiastic about attending workshops and classes dealing with problem solving, the first individual who will need convincing is the principal. Principals are given limited funding and in order to have a successful workshop the principal needs to be reassured that it is money well spent. Principals also need to realize that there is a huge shift happening right now in the area of mathematics and that many of the traditional methods are being replaced with more hands-on strategical operations. Moving away from memorization and calculation means more movement, discussion and also noise. "We know many teachers who get excited about new ways of teaching mathematics by attending courses and workshops, only to return to a wholly, unresponsive principal who neither understands nor values what they are

trying" (Daniels, Hyde, & Zemelman, 1993, p. 81). As an educator, one can certainly request that workshops be attended, but the administrator needs to educate himself or herself about what is best for the school and students. "Administrators should read , go to workshops, and visit classrooms where problem solving is happening, and see for themselves what is possible and what must be done" (Daniels, Hyde, & Zemelman, 1993, p. 81). with the help and backing of the administration, teachers will have a better chance of gaining support form parents who are accustomed to a more traditional means of mathematical instruction.

Effective professional development can provide teachers with the means to engage in exploration, research based inquiry, reflection, experimentation, and practice, while providing collegial sharing of knowledge and opportunities to draw on the expertise of others in the community. Bruining, Spiegel, and Wise (1999), state the following:

> Several factors for delivering effective professional development programs have been identified. They are: providing training, practice, and feedback; providing opportunity for reflection; allowing opportunity for group sharing and inquiry; focusing on student learning an assessment practices; incorporating constructivist approaches to teaching and learning; recognizing teachers as professionals; and providing adequate time and follow-up support. (p.141)

In addition to expanding on their knowledge, "educators should implement their mathematical problem solving program more tightly because it is very easy to fall back on traditional modes of teaching when you try to veer from them" (Frakes & Klein, 2000, p. 378). Teachers should devote time to learn more about how mathematical ideas develop throughout the grades. That way, it will be easier to build the foundations that are necessary for future learning. Beginning at an early age will only improve conditions of mathematical classrooms for upper grade levels. Even at the earliest ages, teachers need to keep in mind that mathematical solutions should not focus on numbers and simply writing the answer, but should instead focus on the method, the process, by which the answer is obtained. The process forms a bridge between the exercises done to learn arithmetic operations and the mathematical methods needed to solve problems reliably. Forming this bridge well before fifth grade is essential to future success in using mathematics (McAllister, 1997).

Teaching Mathematical Vocabulary

Within the realm of teacher collaboration both within and across grade levels is the need to focus on the importance of mathematical vocabulary. In order to develop consistency, educators need to teach mathematical terms, and to ensure that students understand them, before launching into explanations of numerical interactions. "Students at all levels should learn how to use mathematical words or phrases orally before they are expected to represent mathematics symbolically. Just as speaking precedes writing for children, so should the oral language of mathematics precede its symbolization" (Lindquist, Reys, & Suydam, 1992, p. 53). Mathematics has a formal language that students need to understand before they can possibly be expected to execute written explanations of solutions. Also, with learning to speak, it takes time for children to pick up and be able to use language correctly. Children learn words that are repetitive in nature and match their surroundings. Children will then eventually work these words into phrases, and then sentences. It is a growing process. The same is true for the language of mathematics. Students have the capacity to learn these terms, but they also need to be repeated and taught in context in order for a child to retain them. "A child struggling to distinguish between a quotient and a divisor is not ready to understand an explanation of long division" (Cornell, 1999, p.226). Adding meaning to unfamiliar terms and phrases is the key to unlocking the steps for formulating a workable solution. If children can pick out key words when working through story problems, it is much more likely that they will experience success with that particular problem.

During a conference sponsored by the NCTM in Texas, mathematics teachers discovered that a common vocabulary was needed to communicate mathematical questions. The teachers were able to realize this when attempting to explain complex solutions to one another. "When trying out suggested questions for an activity, it was necessary to focus in and talk about specific terms and purposes of the questions" (Chancellor, and Childs, Scheilack, 2000, p. 398). If teachers can use this mathematical language with their students across grade levels, students will be able to travel to various grades with some basic mathematical vocabulary knowledge. Simple problems are excellent for forming reliable problem solving skills. "This is accomplished by understanding the process as well as the vocabulary by which problems are solved" (Cornell, 1999, p. 228).

Fortunately many teachers are becoming aware of the changing mathematical standards and are attempting to reevaluate their current instructional strategies within mathematics. Those

teachers who were using mathematical vocabulary in their daily instruction are beginning to realize that strictly memorizing terms does not lend to comprehension and usage. According to Moore (1995):

> In response to the NCTM standards, many teachers have begun to change math instruction to include oral and written communication . They help students become more comfortable with math language through a variety of strategies. Math literature, manipulatives, games, writing math stories, oral explanations, and math journals are some of the ways students practice math language. (p. 50)

These strategies, some previously mentioned, are all useful ways to incorporate mathematical vocabulary in a fun, non-threatening manner for children. Understanding vocabulary within the context of story problems is key for designing computational strategies and it is often the first step to finding a solution.

To compliment the understanding of mathematical terms and vocabulary words, educators should add mathematical questioning to their language instruction. As important as knowing terms is the ability to explain in a thorough fashion how a solution was obtained. With the new math expectations upon us, the use of mathematical questioning is becoming a necessity. "Teacher must begin to ask students questions such as: How did you find that answer? What was your strategy? Can you show us in a drawing? Can you explain your answer in writing?" (Moore, 1995 p. 52). This type of communication will open doors for exploration and expansion and it will certainly be more interesting for students who are used to a mathematical question and answer format.

Time Allotment for Learning and Reflecting

Another solution that would promote mathematical problem solving is that students need to be allowed more time to think through and reflect on solutions to more complex problems. As stated by Lindquist, Reys, and Suydam (1992):

> Effective teaching of problem solving demands time. Attention must be focused on the relationships in the problem and on the thinking processes involved in reaching a solution. Thus, students must have time to digest or mull over, a problem thoroughly-time to understand the task, time to explore the avenues of the solution, time to think about the solution. Moreover, teachers need to encourage students to extend the amount of time they are willing to work on a problem before giving up. (p. 30)

Today's students live in a world where everything is designed to be fast. Many children do not come into our classrooms ready to spend an extended period of time on one isolated problem. Students are also easily ready to give up if an answer can not be reached with ease. Frakes and Klein (2000) explained the teacher should give students a sufficient amount of 'think time' to solve problems, and squelch the urge to give them more information before they have had time to think, and avoid correcting their mistakes too quickly. Too often teachers provide their students with quick solutions. Teachers also, rely on their quick learners to present complex answers to the class. This can lead to an avoidance of problem solving from those students who do not feel they can solve such problems with such a small time allotment.

Many educators are concerned that using class time for problem solving will decrease the time necessary for another area of instruction. What many teachers don't realize is that mathematical problem solving should not be taught as a separate area of mathematics, but rather in conjunction with the basic computational skills. Some time for problem solving is already included as part of the mathematics curriculum. "Additional time can be gained by organizing instructional activities so that some of the time allotted for practicing computational skills is directed toward problem solving" (Lindquist, Reys, & Suydam, 1992, p. 30). This idea proves to be logical since students who need practice and reinforcement of computational skills can combine this with the practice of solving more complex problems. Students practice both skills, while at the same time use mathematical language and vocabulary within the given assignments.

Another useful strategy that involves additional time is the opportunity to share learned information with peers. "Observing other students solving a problem may help learners internalize either the cognitive functions they are attempting to master or those that are within their zone of proximal development" (Mevarcech, 1999, p. 195). When talking through a problem, students can teach each other while showing that not all solutions mirror one another. This allows children to see that mathematics is multi-faceted and there is often a number of solutions to the same problem.

The solutions presented are precise, but cannot be accomplished without the support and efforts of the educator. What is taught, a child will obtain and believe to be true. Therefore, this reflects the importance of providing time and opportunities for children to explore within their mathematics curriculum.

Reviewing these solutions, the goal of the targeted classrooms is to implement mathematical problem solving strategies that will reflect student improvement within their mathematics environment. The devised plan, which includes a teacher survey, student pretest and posttest, teacher anecdotal notes, and student journal reflections will increase students' mathematical problem solving abilities. Teachers will implement a four step mathematical problem solving method in order to teach a series of problem solving strategies. Throughout the research students will model this four step method through the use of self monitoring checklist, learning logs, journals, and collaboration.

Project Objectives and Processes

As a result of implementing a four step problem solving method during the period of August 2000 to November 2000, the targeted first, third, and sixth grade students will increase problem solving abilities significantly as measured by pretests and posttests.

As a result of daily problem solving exercises during the period of August 2000 to November 2000 the targeted first, third, and sixth grade students will increase their self monitoring abilities as measured by a teacher made checklist

As a result of teaching mathematical vocabulary during the period of August 2000 to November 2000, the targeted first, third and sixth grade students will increase mathematical comprehension by informal teacher observation and pretests and posttests.

In order to accomplish the objectives, the following processes are necessary:

1. Administer a teacher made survey.
2. Introduce a four-step problem solving method.
3. Teach a variety of problem solving applications and skills.
4. Monitor understanding through the use of checklists and self-monitoring reflections.

Action Plan

<u>Week #1 August 28, 2000</u>

- The teacher researchers will administer a teacher survey.
- All will pass out parent permission letter to conduct research.

<u>Week #2 September 5, 2000</u>

- All will administer a mathematical problem solving pretest.
- All will begin instruction on a four-step method for problem solving.
- All will post a chart listing a four-step method for problem solving.
- Students will write an initial journal reflection.

Week #3 September 11, 2000

- Introduce problem solving skill: Look for a pattern.
- All give direct instruction each day using a different practice problem relating to the skill.
- Following, small groups will collaborate and solve a similar problem using a four-step method.
- These problems will then be shared during a whole group discussion.
- At the end of the week, students will complete a mathematical learning log regarding the required skill.
- All will post a chart illustrating "Look for a pattern".

Week #4 September 18, 2000

- Introduce problem solving skill: Make an organized list.
- All give direct instruction each day using a different practice problem relating to the skill
- Following, small groups will collaborate and solve a similar problem using a four-step method.
- These problems will then be shared during a whole group discussion.
- At the end of the week, students will complete a mathematical learning log regarding the required skill.
- All will post a chart illustrating "Make an organized list".

Week #5 September 25, 2000

- Introduce problem solving skill: Make a table or chart.
- All give direct instruction each day using a different practice problem relating to the skill
- Following, small groups will collaborate and solve a similar problem using a four-step method.
- These problems will then be shared during a whole group discussion.
- At the end of the week, students will complete a mathematical learning log regarding the required skill.
- All will post a chart illustrating "Make a table or chart".
- Student will write a journal reflection.

Week #6 October 2, 2000

- Introduce problem solving skill: Guess and check.
- All give direct instruction each day using a different practice problem relating to the skill
- Following, small groups will collaborate and solve a similar problem using a four-step method.
- These problems will then be shared during a whole group discussion.
- At the end of the week, students will complete a mathematical learning log regarding the required skill.
- All will post a chart illustrating "Guess and check".

Week #7 October 16, 2000

- Introduce problem solving skill: Work backward.
- All give direct instruction each day using a different practice problem relating to the skill
- Following, small groups will collaborate and solve a similar problem using a four-step method.
- These problems will then be shared during a whole group discussion.
- At the end of the week, students will complete a mathematical learning log regarding the required skill.
- All will post a chart illustrating "Work backward".

Week #8 October 23, 2000

- All will conduct a review of the strategies taught.
- Teachers will use daily sample problems.
- Students will solve and explain the sample problems using the four-step checklist.

Week #9 October 30, 2000

- All will administer a mathematical problem solving post-test.
- Students will be asked to reflect on their progress in their journals.
- Students will write a final journal reflection.

Methods of Assessment

To measure the effectiveness of the action research project, a student pretest and posttest will be administered to calculate growth over a nine week period. As the project develops, the level of mastery should improve as a result of the implementation of the action plan. A student reflection journal will also be written periodically to summarize the childrens' thoughts and attitudes about mathematical problem solving. While teaching a variety of problem solving strategies, teachers will also encourage the students to implement a checklist for their story problems. This student checklist will encourage students to follow a four-step problem solving method rather than simply focusing on basic computational concepts. From these data, the researchers will compare before and after results.

CHAPTER 4

PROJECT RESULTS

Historical Description of the Intervention

The terminal objective of the intervention was to address the inadequate mathematical problem solving abilities of students that, in turn, interfered with students' abilities to plan and use complex mathematical strategies. Indications were from teacher observations, lack of cohesiveness throughout grade levels, and the absence of a self-monitoring device.

During the first week of the school year a teacher survey was administered in order to gain insight on the beliefs and routines of mathematics educators (Appendix B). Along with this, parents were also notified of the upcoming research through a parent permission form (Appendix A). Once permission for research was established, students were given a mathematical problem solving pretest consisting of five general problem solving areas (Appendix C). The areas were as follows: Look for a Pattern, Organized Lists, Making a Table or Chart, Guessing and Checking, and Working Backwards. To also gain an understanding of what the students perceived problem solving to be, students were asked to complete an introductory journal reflection (Appendix D). The journal asked students to establish the meaning of mathematical problem solving and to explain their own procedures for solving a problem.

Following initial surveys, testing, and journals, students were exposed to and instructed on a four-step method for problem solving. Using various examples, students practiced the process of devising and executing a problem solving plan. This four-step problem solving method was then posted within the classroom as a daily reminder. students were then asked to use a problem solving checklist to monitor their work (Appendix E).

The lessons for the research were divided into 5 consecutive weeks. Each week focused on a specific problem solving strategy. On the first day of the week the strategy was introduced to the students. Direct instruction was provided to the entire group using the four-step method. Each day of the week following focused on a similar problem relating to the same strategy being focused

on. Small groups consisting of three to four students collaborated and solving a problem each day using the four-step method. Groups were also instructed to use their checklist as a self-monitoring guide. The problems were then discussed as a whole class, giving small groups an opportunity to share and also make corrections. At the close of the week, the strategy was reviewed a final time and students recorded solutions in a mathematical learning log. A chart was also posted to remind students of the strategy. Each chart reviewed a specific problem and gave examples of how to solve it.

After the 5 week duration, a week of review was administered. A daily problem was posted and students attempted to plan and solve the problems independently. These problems reflected the five strategies covered during the course of the research.

To monitor student progress and understanding throughout the mathematical problem solving process, students recorded solutions in a notebook specifically used for their daily math problems. This was labeled as a Learning Log. Along with this, students were also asked to record journal reflections before, during, and following the course of the research. The first journal reflection was previously established. The second journal asked students to reflect on the four-step method as to whether or not they found it beneficial (Appendix F). The final journal reflection asked the students to record how or if their ideas about mathematical problem solving have changed based upon the work completed in the previous weeks (Appendix G). As closure, students were then administered a mathematical problem solving posttest, which was identical to the pretest (Appendix H).

Presentation and Analysis of Project Results

In order to assess the effect of the mathematical problem solving instruction of the targeted classes, the researchers kept anecdotal records of group activities, analyzed the responses of student journals, as well as, analyzed the results of an administered pretest and posttest, all of which were given before and after the implementation of the instruction. The results of the posttest are presented in Figure 4.

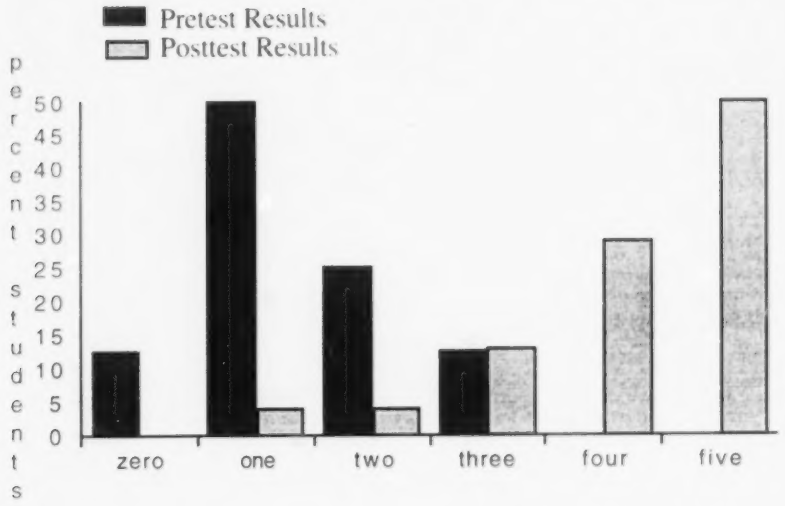

Figure 4. Site A pretest results compared to posttest results of percent of students and number of correct answers on problem solving task.

Figure 4 clearly demonstrates an increase in student achievement after the previously stated plans were implemented. Half of the students achieved five correct answers on the posttest, while only a few students answered two or less questions correctly.

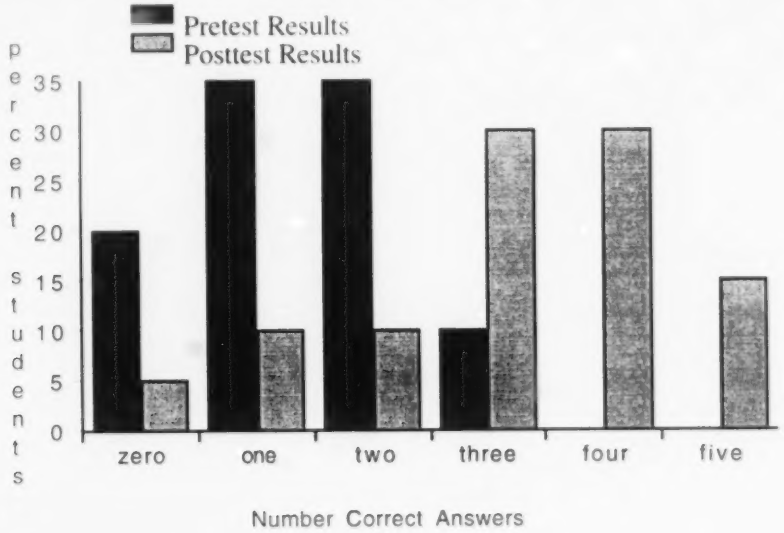

Figure 5. Site O pretest results compared to posttest results of percent of students and number of correct answers on problem solving task.

The results in Figure 5 show 75% of the students answered three or more questions correctly. Figure 5 suggests an increasing in the understanding of the use of mathematical problem solving strategies.

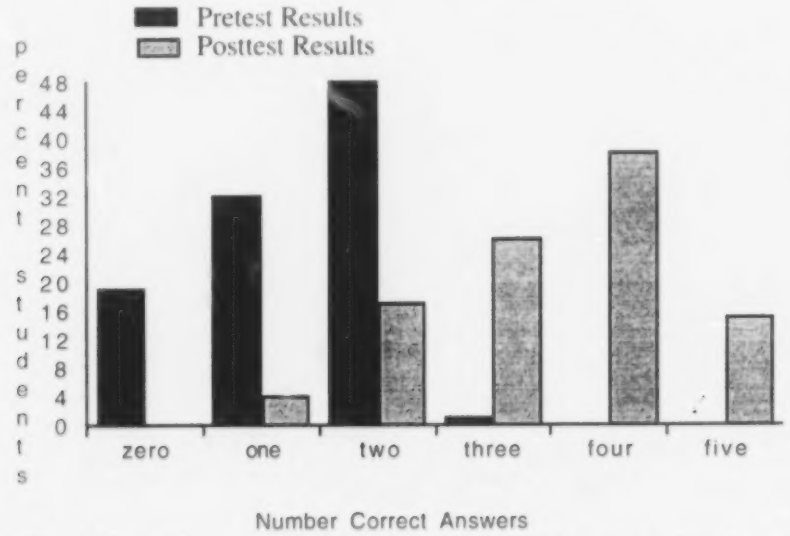

Figure 6. Site W pretest results compared to posttest results of percent of students and number of correct answers on problem solving task.

Finally, Figure 4 corresponds with the results shown in Figures 5 and 6. More than half of the students answered three or more questions correctly.

Generally, the posttests indicated that more than half of the students improved their ability to successfully implement mathematical problem solving strategies. However, the results in Figures 4, 5, and 6 indicate that additional implementation of mathematical problem solving strategies were still needed.

Along with the pretest and posttest, the researchers reviewed student journal reflections in order to evaluate growth and change. In the initial journal, students were asked to define problem solving and explain how they solve a problem. At the first grade level, students had little to no understanding of the questions. With writing being difficult as well, their responses were inconclusive at that time. The most common response for third graders was finding an answer to a question. None of the third grade students discussed using steps or a plan. In sixth grade, the responses also included finding an answer to a question. However, at this level many students were able to express the need to work through steps.

The second journal entry asked the students to explain whether the problem solving method was beneficial. In almost every instance, at all three grade levels, students agreed that using a plan was helpful. The self-monitoring checklist also proved to be a positive tool within this journal entry for many of the students in third and six grade. First graders did not mention the checklist as frequently.

The final journal reflection asked the students to record their growth over the period of research. Again, there was an extensive amount of positive responses. Students generally found the plan to be helpful and they enjoyed having the chance to solve problems with a group. The only negative factor that a number of sixth graders mentioned was the time factor. They felt that these problems were often time consuming and at times cumbersome.

The researchers took anecdotal notes documenting the progress of the students during the mathematical problem solving learning activities (Appendix I). Initially, as the four-step method and self-monitoring checklist were introduced, a majority of the students experienced confusion and frustration. In the first few lessons, most groups needed reminders and prompting in order to get them to use the checklist. Another early observation was that when students were asked verbally to explain items on the checklist, a number of students still lacked understanding. First grade students struggled a great deal with the checklist and it was noted that visual cues may have been a help for them.

As activities finalized, researchers noted that the majority of the students demonstrated a general understanding of the four step method as well as the checklist. While many of the daily problems presented challenges, students were much more willing to attempt a solution.

Conclusions and Recommendations

The results of the action plan showed favorable results for improving students' mathematical problem solving ability through the use of planning and self-monitoring. Research has shown that method rather than result is what many children are lacking within their mathematics curriculum. It is evident that our daily math practices need to evolve in order to meet today's mathematical testing standards. Students will benefit educationally through the use of mathematical problem solving.

Based on the results of the research project, the researchers believe the following mathematical problem solving components need to be emphasized within a daily mathematics

curriculum. They are listed as follows: (a) provide a problem each day that will develop problem solving strategies, (b) use and develop a method for solving problems, (c) provide time for students to collaborate, (d) allow for a self-monitoring device, (e) encourage the use of learning logs and allow for reflection.

The information generated from this action research project indicated favorable response to teaching mathematical problem solving strategies in schools today. Lack of cohesiveness throughout grade levels, poor vocabulary , and a misunderstanding of what mathematical problem solving is have all contributed to a decrease in the appropriate problem solving methods that students need. This research has shown that it is imperative that educators work together to create mathematical problem solving structure for students.

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Appendices

Appendix A

ST. XAVIER UNIVERSITY CONSENT TO PARTICIPATE IN RESEARCH STUDY ENTITLED
BEYOND COMPUTATION: IMPROVING MATHEMATICAL PROBLEM SOLVING

Dear parent/guardian,

I'm presently working on my Master's of Arts in Teaching and Leadership degree at St. Xavier's University. As a candidate for the master's program I will be conducting an educational research project. The purpose of the research project is to improve mathematical problem solving skills.

Your child will participate in a series of problem solving activities. I will also be administering a pre and post-test in order to track progress. These tests will simply be used to track problem solving progress and will not count against your child's grade.

The benefits of this research study consist of improving mathematical problem solving skills and greater preparation for the future. Participation is completely voluntary. Student's name and program results will not be released. I am only interested in seeing how to provide your child with the best education. If you have any questions please feel free to contact me. Thank you for your cooperation.

Sincerely,

Please sign and return the bottom portion of this consent form by _____

I, the parent/legal guardian of _____ acknowledge that the teacher has explained to me the need for this research, explained what is involved and offered to answer any questions. I freely and voluntarily consent to my child's participation in this study. I understand all information gathered during the study will be completely confidential.

Name of student _____

Signature of parent/legal guardian _____ Date _____

Appendix B

Mathematical Problem Solving
Teacher Survey

Dear Colleagues,

I am currently pursuing my Master's Degree and I would greatly appreciate your input on my research. The topic of my research is Mathematical Problem Solving. I am studying some ways that both students and teachers can successfully implement problem solving strategies within the mathematics curriculum. Please take a few moments to complete this brief survey and place it in my mailbox by _____.
Thank you in advance for your cooperation and support!

Sincerely,

Circle 1 if your answer is NEVER.
Circle 2 if your answer is SELDOM.
Circle 3 is your answer is SOMETIMES.
Circle 4 if your answer is OFTEN.
Circle 5 is your answer if ALWAYS.

1. How often do your students demonstrate correct usage and understanding of math vocabulary?	1	2	3	4	5
2. How often do you use problems that promote real world application?	1	2	3	4	5
3. Do you provide a self-monitoring checklist for your students when problem solving?	1	2	3	4	5
4. Do you provide opportunities for group discussions and interactions during mathematics?	1	2	3	4	5
5. Do you focus your teaching on mastery and memorization of key concepts?	1	2	3	4	5
6. Do your students use a variety of applications when solving a problem?	1	2	3	4	5
7. How often are your students expected to plan a mathematical solution either verbally or written?	1	2	3	4	5

- OVER -

Please check yes or no for each of the following items.

_____yes _____no 1. Time is a concern when teaching problem solving.

_____yes _____no 2. My math curriculum provides enough problem solving materials.

_____yes _____no 3. My students keep a mathematics journal for reflections.

_____yes _____no 4. Teachers would benefit from workshops pertaining to problem solving.

List a few things you are currently doing to promote problem solving in your classroom.

_____

_____

_____

_____

_____

_____

Appendix C

Name_____

Date_____

Problem Solving Pre-Test

1.

What would come next in the pattern?

_____

2. Carlos wants to buy exactly 7

Which should he buy?

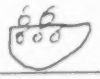
Guess and check.

Circle the correct to show your answer.

3. Jill has a choice of 3 different colors of balloons.
Her mom said she can only pick 2 balloons.
How many different groups of 2 balloons can she pick?
She can pick from red, orange or green.

4. Jill wants to fill 6 bowls.
   She wants 5 apples in each bowl.
   How many apples does Jill need?
   Finish the table.

Apples Jill Needs

Number of Bowls						
Apples	5	10	_____	_____	_____	_____

Jill needs_____apples.

5. Jack ran out of his house to get to his baseball game.
   He ran 3 blocks and realized he forgot his mitt so he had to run back. What block is Jack on if he lives on block 4.

_____

Name_____Date_____

Problem Solving Pre-Test

Directions: Solve each problem in the space provided. Show your work. After each problem write the strategy that you used.

1. Complete the table.

In	3	4	5		
Out	9	10	11		

Strategy Used_____

2. A clown has three wigs. One wig is black, one is red, and one is green. The clown also has three pairs of glasses. One pair is yellow, one is blue, and one is white. List all the combinations of wigs and glasses that can be worn.

Strategy Used_____

3. Julie is making a necklace that will spell her name in letter beads. Between each letter, she will put 5 red beads. She will also put 5 red beads at each end of the name. How many red beads will she need for the necklace?

Strategy Used_____

4. The sum of two numbers is 55. The numbers are 7 apart. What are they?

Strategy Used_____

5. Mark looks at the calendar on his birthday and says, "The carnival is only three weeks and two days away!" If the carnival is on Monday, May 30th, what day and date is Mark's birthday?

Strategy Used_____

Name _____

Date _____ Hour _____

Problem Solving Pre-Test

Directions: Solve the following problems. Show your work on a separate piece of lined paper. After each problem name the "Problem Solving Strategy" you used.

1. During their first year of life, swordfish increase in weight at a regular rate. A swordfish weighed 14 pounds at age 1 month and 28 pounds at age 2 months. How much did it weigh at age 6 months?

Strategy Used _____

2. There are 5 pitchers and 3 catchers on the Century Wildcat baseball team. How many pitcher-catcher pairs can coach Olson choose from?

Strategy Used _____

3. Every student at Century has 2 parents, 4 grandparents, 8 great-grandparents, and so on. Record this information in a table. Then find how many great-great-great-great-great-great grandparents everyone has.

Strategy Used _____

4. Before going on Vacation, Jenny bought 21 rolls of film. She bought twice as many rolls of print film as slide film. How many rolls of each type did she buy?

Strategy Used _____

5. One winter night, the temperature in Orland Park fell 14 degrees between midnight and 6 A.M. Between 6 A.M. and 10 A.M., the temperature doubled. By noon it had risen another 11 degrees, to 33° F. Find the midnight temperature.

Strategy Used _____

What is problem solving in math?

How do you solve math problems?

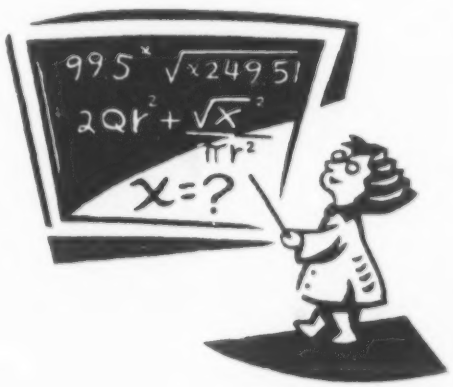
Appendix E

Use the following four steps to guide you through solving a problem.

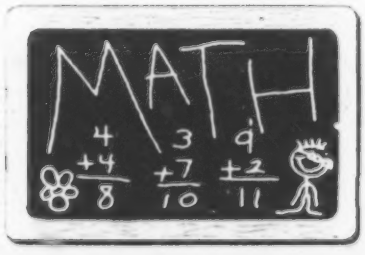
Student Problem Solving Checklist

	Yes	No
Do I understand the problem? What do I know? What do I need to find out?		
Did I develop a plan? Have I ever solved a similar problem? What strategies can I use? Now I can estimate an answer.		
Now I can solve the problem. Do I need to try another strategy? What is the solution?		
Look back. Did I answer the right question? Does my answer make sense?		

Has the use of the four-step method helped you solve problems? Explain.



Have your ideas about mathematical problem solving changed? List your likes and dislikes.



Appendix H

Name_____

Date_____

Problem Solving Post-Test

1.

What would come next in the pattern?

_____

2. Carlos wants to buy exactly 7

Which should he buy?

Guess and check.

Circle the correct to show your answer.

3. Jill has a choice of 3 different colors of balloons.
Her mom said she can only pick 2 balloons.
How many different groups of 2 balloons can she pick?
She can pick from red, orange or green.

4. Jill wants to fill 6 bowls.
   She wants 5 apples in each bowl.
   How many apples does Jill need?
   Finish the table.

Apples Jill Needs

Number of Bowls						
Apples	5	10	_____	_____	_____	_____

Jill needs_____apples.

5. Jack ran out of his house to get to his baseball game.
   He ran 3 blocks and realized he forgot his mitt so he had to run back. What block is Jack on if he lives on block 4.

_____

Name_____ Date_____

Problem Solving Post-Test

Directions: Solve each problem in the space provided. Show your work. After each problem write the strategy that you used.

1. Complete the table.

In	3	4	5		
Out	9	10	11		

Strategy Used_____

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Strategy Used_____

3. Julie is making a necklace that will spell her name in letter beads. Between each letter, she will put 5 red beads. She will also put 5 red beads at each end of the name. How many red beads will she need for the necklace?

Strategy Used_____

4. The sum of two numbers is 55. The numbers are 7 apart. What are they?

Strategy Used_____

5. Mark looks at the calendar on his birthday and says, "The carnival is only three weeks and two days away!" If the carnival is on Monday, May 30th, what day and date is Mark's birthday?

Strategy Used_____

Name _____

Date _____ Hour _____

Problem Solving Post-Test

Directions: Solve the following problems. Show your work on a separate piece of lined paper. After each problem name the "Problem Solving Strategy" you used.

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Strategy Used _____

2. There are 5 pitchers and 3 catchers on the Century Wildcat baseball team. How many pitcher-catcher pairs can coach Olson choose from?

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3. Every student at Century has 2 parents, 4 grandparents, 8 great-grandparents, and so on. Record this information in a table. Then find how many great-great-great-great-great-great grandparents everyone has.

Strategy Used _____

4. Before going on Vacation, Jenny bought 21 rolls of film. She bought twice as many rolls of print film as slide film. How many rolls of each type did she buy?

Strategy Used _____

5. One winter night, the temperature in Orland Park fell 14 degrees between midnight and 6 A.M. Between 6 A.M. and 10 A.M., the temperature doubled. By noon it had risen another 11 degrees, to 33° F. Find the midnight temperature.

Strategy Used _____